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$D^{*\pm}_{s0}(2317)$ and KD scattering from B^0_s decay

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ABSTRACT

We study the $\bar{B}^0_s \to D^-_s (KD)^+$ weak decay, and look at the KD invariant mass distribution, for which we use recent lattice QCD results for the KD interaction from where the $D^*_{s0}(2317)$ resonance appears as a KD bound state. Since there are not yet experimental data on this reaction, in a second step we propose an analysis method to obtain information on the $D^*_{s0}(2317)$ resonance from the future experimental KD mass distribution in this decay. For this purpose, we generate synthetic data taking a few points from our theoretical distribution, to which we add a 5% or 10% error. With this analysis method, we prove that one can obtain from these "data" the existence of a bound KD state, the KD scattering length and effective range, and most importantly, the KD probability in the wave function of the bound state obtained, which was found to be largely dominant in lattice QCD studies. This means that one can obtain information on the nature of the $D^{*+}_{s0}(2317)$ resonance from the implementation of this experiment, in the line of finding the structure of resonances, which is one of the main aims in hadron spectroscopy.

1. Introduction

The very narrow charmed scalar meson $D^{*+}_{s0}(2317)$ was first observed in the $D^+_s\pi^0$ channel by the BABAR Collaboration [1] and its existence was confirmed by CLEO [2], BELLE [3] and FOCUS [4] Collaborations. Its mass was commonly measured as 2317 MeV, which is approximately 160 MeV below the prediction of the very successful quark model for the charmed mesons [5]. Due to its low mass, the structure of the meson $D^{*\pm}_{s0}(2317)$ has been extensively debated. It has been interpreted as a $c\bar{s}$ state [6–10], two-meson molecular state [11–20], $K-D$-mixing [21], four-quark states [22–25] or a mixture between two-meson and four-quark states [26]. Within the molecular interpretations we mention in particular Ref. [19], where it is shown that the presence of u and d quarks in the D and K mesons gives rise to a strong isospin violation responsible for the $D^*_{s0} \to D_s\pi$ transition, which gives rise to the D^*_{s0} width. Additional support to the molecular interpretation came recently from lattice QCD simulations [27–30]. In previous lattice studies of the $D^*_{s0}(2317)$, it was treated as a conventional quark–antiquark state and no states with the correct mass (below the KD threshold) were found. In Refs. [27,29], with the introduction of KD meson operators and using the effective range formula, a bound state is obtained about 40 MeV below the KD threshold. The fact that the bound state appears with the KD interpolator may be interpreted as a possible KD molecular structure, but more precise statements cannot be done. In Ref. [28] lattice QCD results for the KD scattering length are extrapolated to physical pion masses by means of unitarized chiral perturbation theory, and by means of the Weinberg compositeness condition [31,32] the amount of KD content in the $D^*_{s0}(2317)$ is determined, resulting in a sizable fraction of the order of 70% within errors. A reanalysis of the lattice spectra of Refs. [27,29] has been recently done in Ref. [30], going beyond the effective range approximation and making use of the three levels of Refs. [27,29]. As a consequence, one can be more quantitative about the nature of the $D_{s0}(2317)$, which appears with a KD component of about 70%, within errors.

In addition to these lattice results, and more precise ones that should be available in the future, it is very important to have some experimental data that could be used to test the internal structure of this exotic state.

Here we propose to use the experimental KD invariant mass distribution of the weak decay of $\bar{B}^0_s \to D^-_s(DK)^{+1}$ in order to obtain information about the internal structure of the $D^{*+}_{s0}(2317)$ state. It is worth mentioning that previous work already exists in the study of B decays into a heavy meson and a molecular mesonic

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[1] Throughout this work, the notation $(DK)^+$ refers to the isoscalar combination $D^0K^+ + D^+K^0$.

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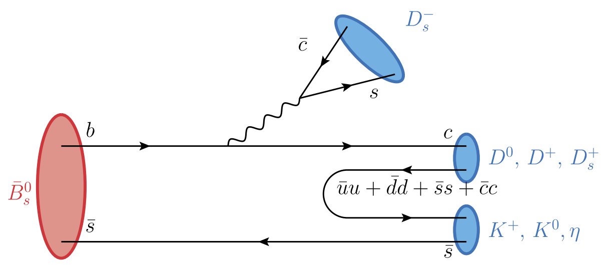

Fig. 1. Mechanism for the decay $\bar{B}_s^0 \to D_s^-(DK)^+$.

state, such as Ref. [20], where the reactions $B \to D^{(*)} D_{s0}^*(D_{s1})$ are studied, although the purpose, formalism and reaction are different from those in this work. There are not yet experimental data for the decay $\bar{B}_s^0 \to D_s^-(DK)^+$. However, since the branching fractions for the decays $\bar{B}_s^0 \to D_s^{*+} D_s^{*-}$ and $\bar{B}_s^0 \to D_s^+ D_s^{*-} + D_s^{*+} D_s^-$ are respectively 1.85% and 1.28%, we believe that the branching fraction for the $\bar{B}_s^0 \to D_s^- D_{s0}^{*+}$ decay, should not be so different from that and it will be seen through the channel $\bar{B}_s^0 \to D_s^-(DK)^+$. This is why it is really important to have theoretical predictions for the DK invariant mass distribution that considers the formation of the $D_{s0}^{*+}(2317)$ state. At this point, it is worth stressing that recently, in the reactions $B^0 \to D^- D^0 K^+$ and $B^+ \to \bar{D}^0 D^0 K^+$ studied by the BABAR Collaboration [33], an enhancement in the invariant DK mass in the range $2.35 - 2.50$ GeV is observed, which could be associated with this $D_{s0}^{*+}(2317)$ state. It is also interesting to quote that in a different reaction, $B_s^0 \to \bar{D}^0 K^- \pi^+$, the LHCb Collaboration also finds an enhancement close to the KD threshold in the $\bar{D}^0 K^-$ invariant mass distribution, which is partly associated to the $D_{s0}^*(2317)$ resonance [34]. Concerning the present reaction, the LHCb Collaboration is working on it, and results are expected probably within one year.[2]

In Fig. 1 we show the mechanism for the decay $\bar{B}_s^0 \to D_s^-(DK)^+$. The idea is to take the dominant mechanism for the weak decay of the $\bar{B}_s^0$ into D_s^- plus a primary $c\bar{s}$ pair. The hadronization of the initial $c\bar{s}$ pair is achieved by inserting a $q\bar{q}$ pair with the quantum numbers of the vacuum: $u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c}$, as shown in Fig. 1. Therefore, the $c\bar{s}$ pair is hadronized into a pair of pseudoscalar mesons. This pair of pseudoscalar mesons is then allowed to interact to produce the $D_{s0}^{*+}(2317)$ resonance, which is considered here as mainly a DK molecule [15]. The idea is similar to the one used in Ref. [35] for the formation of the $f_0(980)$ and $f_0(500)$ scalar resonances in the decays of B^0 and B_s^0.

The paper is organized as follows. In Section 2 we settle the formalism for our study. Namely, in Subsection 2.1 we study the $(DK)^+$ elastic scattering amplitude, and in Subsection 2.2 we study the differential decay width for the process $B_s^0 \to D_s^-(DK)^+$. As said before, there is not yet experimental information concerning the differential decay width for this process. For this reason, we will have to generate synthetic data for this decay in order to explore if this reaction is suitable for the study of the $(DK)^+$ final state interactions and the $D_{s0}^{*+}(2317)$ bound state. The generation and analysis of these synthetic data, which constitutes the results of the work, are done in Section 3. Conclusions are delivered in Section 4.

2. Formalism

In this work the influence of the presence of the $D_{s0}^{*+}(2317)$ in the process $\bar{B}_s^0 \to D_s^-(DK)^+$ is investigated. The $D_{s0}^{*+}(2317)$ is considered mainly as a bound state of the DK system, so we address the elastic DK scattering amplitude in Subsection 2.1. Then, the differential decay width for the $\bar{B}_s^0 \to D_s^-(DK)^+$ reaction in terms of the DK invariant mass is considered in Subsection 2.2.

2.1. Elastic DK scattering amplitude

Let us start by discussing the S-wave amplitude for the isospin $I = 0$ DK elastic scattering, which we denote T. It can be written as [36]:

$$T^{-1}(s) = V^{-1}(s) - G(s) \Rightarrow T(s) = V(s)(1 + G(s)T(s)) \,, \tag{1}$$

where $G(s)$ is a loop function bearing the unitary or right hand cut,

$$G(s) \equiv i \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m_K^2 + i\epsilon} \frac{1}{(q-l)^2 - m_D^2 + i\epsilon}, \tag{2}$$

and $s = q^2$ is the invariant mass squared of the DK system. This function needs to be regularized, and this is accomplished in this work by means of a subtraction constant, $a(\mu)$. In this way, the G function can be written as [36]:

$$\begin{aligned} 16\pi^2 G(s) &= a(\mu) + \log \frac{m_D m_K}{\mu^2} + \frac{\Delta}{2s} \log \frac{m_D^2}{m_K^2} \\ &\quad + \frac{\nu}{2s}\left(\log \frac{s - \Delta + \nu}{-s + \Delta + \nu} + \log \frac{s + \Delta + \nu}{-s - \Delta + \nu} \right) , \\ \Delta &= m_D^2 - m_K^2 \,, \quad \nu = \lambda^{1/2}(s, m_D^2, m_K^2) \,, \end{aligned} \tag{3}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ is the Källen or triangle function. In Eq. (1), $V(s)$ is the potential, typically extracted from some effective field theory, although a different approach will be followed here (see below).

The amplitude $T(s)$ can also be written in terms of the phase shift $\delta(s)$ and/or effective range expansion parameters [37],

$$T(s) = -\frac{8\pi\sqrt{s}}{p_K \,\mathrm{ctg}\delta - i p_K} \simeq -\frac{8\pi\sqrt{s}}{\frac{1}{a} + \frac{1}{2} r_0 p_K^2 - i p_K}, \tag{4}$$

with

$$p_K(s) = \frac{\lambda^{1/2}(s, M_K^2, M_D^2)}{2\sqrt{s}} \,, \tag{5}$$

the momentum of the K meson in the DK center of mass system. Above, a and r_0 are the scattering length and the effective range, respectively.

In this channel and linked to it we find the $D_{s0}^{*+}(2317)$ resonance, the object of study of this paper, below the DK threshold, the latter being located roughly above 2360 MeV. This means that the amplitude has a pole at the squared mass of this state, $M^2 \equiv s_0$, so that, around the pole,

$$T(s) = \frac{g^2}{s - s_0} + \text{regular terms}, \tag{6}$$

being g the coupling of the state to the DK channel. From Eqs. (1) and (6), we see that (the following derivatives are meant to be calculated at $s = s_0$):

$$\frac{1}{g^2} = \frac{\partial T^{-1}(s)}{\partial s} = \frac{\partial V^{-1}(s)}{\partial s} - \frac{\partial G(s)}{\partial s} \,. \tag{7}$$

We have thus the following exact sum rule,

$$1 = g^2 \left(-\frac{\partial G(s)}{\partial s} + \frac{\partial V^{-1}(s)}{\partial s} \right) . \tag{8}$$

[2] T. Gershon, private communication.

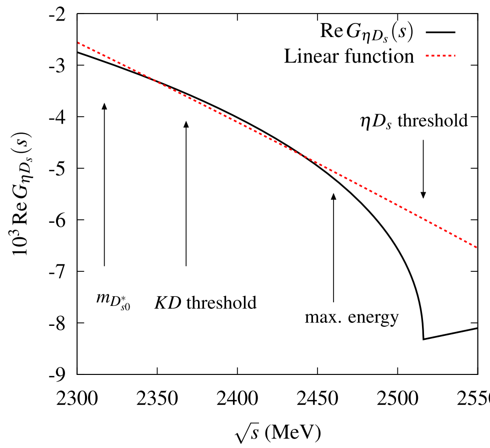

Fig. 2. Real part of the loop function $G(s)$ for the ηD_s channel in Eq. (15) (black solid line) compared with a function linear in s (red dashed line) that approximates the former in the energy range considered in this work. Both functions are shown in a larger range in order to check the accuracy of the approximation.

In Ref. [38] it has been shown, as a generalization of the Weinberg compositeness condition [31] (see also Ref. [39] and references therein), that the probability P of finding the channel under study (in this case, DK) in the wave function of the bound state is given by:

$$P = -g^2 \frac{\partial G(s)}{\partial s} , \tag{9}$$

while the rest of the r.h.s. of Eq. (8) represents the probability of other channels, and hence the probabilities add up to 1. Finally, if one has an energy independent potential the second term of Eq. (8) vanishes, and then $P = 1$, that is, the bound state is purely given by the channel under consideration. In Ref. [38], these ideas are generalized to the coupled channels case.

Let us now apply these ideas to the case of DK scattering. From Eq. (1) it can be seen that the existence of a pole implies

$$V^{-1}(s) \simeq G(s_0) + \alpha(s - s_0) + \cdots , \tag{10}$$

$$\alpha \equiv \left.\frac{\partial V^{-1}(s)}{\partial s}\right|_{s=s_0} , \tag{11}$$

in the neighborhood of the pole. If we assume that the energy dependence is smooth enough to allow us to retain only the linear term in s, we can now write the amplitude as

$$T^{-1}(s) = G(s_0) - G(s) + \alpha(s - s_0) , \tag{12}$$

and the sum rule in Eq. (8) becomes:

$$P_{DK} = 1 - \alpha g^2 . \tag{13}$$

In this way, the quantity αg^2 represents the probability of finding other components beyond DK in the wave function of $D^{*+}_{s0}(2317)$. The following relation can also be deduced from Eqs. (13) and (9):

$$\alpha = -\frac{1 - P_{DK}}{P_{DK}} \left.\frac{\partial G(s)}{\partial s}\right|_{s=s_0} . \tag{14}$$

To justify keeping just the linear term in Eq. (10) let us see which is the origin of the energy in the potential from a physical point of view. In Ref. [15] it was seen that the DK and $D_s\eta$ channels were the building blocks of the $D^*_{s0}(2317)$ state, and a coupled channel analysis was performed. It is possible to eliminate the $D_s\eta$ channel and perform a study with only the DK channel, at the expense of using an effective KD potential which is, however, energy dependent [39,40]. In Ref. [41], one can see explicitly for the case of two channels that the effective potential is:

$$V^{(\mathrm{eff})}_{KD,KD}(s) = V_{KD,KD} + V^2_{KD,\eta D_s} G_{\eta D_s}(s) , \tag{15}$$

where $V_{KD,KD}$ and $V_{KD,\eta D_s}$ are the diagonal KD and the transition $KD \to \eta D_s$ original potentials, and $G_{\eta D_s}(s)$ is the loop function for the ηD_s two-meson system. In chiral theories, the potentials computed from the Lagrangians usually have a term which is linear in s, but there are others which are constant. Much of the energy dependence in the effective potential of Eq. (15) comes, however, from the loop function of the channel that has been eliminated (ηD_s). For the case of KD and ηD_s channels, this second loop function has a discontinuity in the derivative at the ηD_s threshold. But it is important to notice that this threshold is located around 200 MeV above the mass of the $D^*_{s0}(2317)$, and from this mass to the maximum energy considered in this work (2460 MeV) a linear approximation for it is excellent. This can be seen in Fig. 2, where the real part of the exact $G_{\eta D_s}(s)$ function (black solid line) is shown together with its linear (in s) approximation (red dashed line). Regarding the energy dependence of the potential itself, as said, they usually have a linear term in s when computed to lowest order in different effective field theories. However, this dependence is usually smooth enough so that the inverse of the potential can also be expanded up to a linear term in the small range we are considering in this work. Specifically, this is the case for the interaction potential of the channel under study, $KD \to KD$, in the hidden gauge formalism in Ref. [15].

In principle, one can also have in $V(s)$ terms like $\beta/(s - s_R)$, known as Castillejo–Dalitz–Dyson (CDD) poles [42]. These poles (or the associated ones in T^∞ of Ref. [43]) are often related [39,40,43,44] to the possible existence of states of non-molecular nature, or genuine states, or Adler zeroes of the amplitudes. In the present case we can use the results of the lattice spectra study of Ref. [30]. Quoting textually from this latter work, "the statistics of the obtained fits shows a clear preference for solutions with a s_R value that lies far away (more than 300 MeV) from the KD threshold, such that it effectively provides a linear dependence on $s - s_{\mathrm{th}}$ at the energies where the pole is found."

Additionally, one has a left hand cut originating from 2π exchange and starting at $\vec{p}^{\,2}_K = -m^2_\pi$, with m_π the pion mass. This branching point is located between the threshold and the position of the $D^*_{s0}(2317)$ pole. This contribution could provide an energy dependence different than the linear one. Once again, we can rely on previous works [45–47] that compute the two-pion exchange potential in similar cases, and which have shown that this source of interaction is extremely weak compared with the ordinary one from vector exchange, so that it can be safely neglected.

In short, we have seen that all possible contributions to the inverse of the potential in Eq. (10) can be expanded linearly and hence this assumption, leading to Eq. (12), is quite reasonable.

We can now link this formalism with the results of Ref. [30], where a reanalysis is done of the energy levels found in the lattice simulations of Ref. [29]. In Ref. [30], the following values for the effective range parameters are found:

$$a_0 = -1.4 \pm 0.6 \text{ fm} , \quad r_0 = -0.1 \pm 0.2 \text{ fm} . \tag{16}$$

Also, in studying the $D^{*+}_{s0}(2317)$ bound state, a binding energy $B = M_D + M_K - M_{D^{*+}_{s0}} = 31 \pm 17$ MeV is found in Ref. [30]. The probability P_{DK} is also studied, and the value $P_{DK} = 0.72 \pm 0.12$ is found. Hence, for our analysis, in which synthetic data for the reaction $\bar{B}^0_s \to (DK)^+ D^-_s$ will be generated, we can start from the hypothesis that a bound state exists in the DK channel, with a

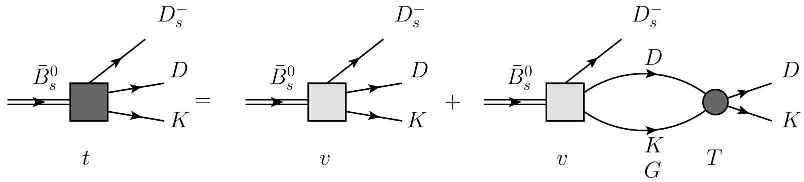

Fig. 3. Diagrammatical interpretation of Eq. (21), in which DK final state interaction is taken into account for the decay $\bar{B}_s^0 \to D_s^-(DK)^+$. The dark square represents the amplitude t for the process, in which the final state interaction is already taken into account. The light square represents the bare vertex for the process, denoted by v. Finally, the circle represents the hadronic amplitude for the elastic DK scattering.

mass $M_{D_{s0}^{*+}} = 2317$ MeV (the nominal one), and with a probability $P_{DK} = 0.75$. This implies, from Eq. (14), the value $\alpha = 2.06 \cdot 10^{-3}$ GeV^{-2}. Finally, for the subtraction constant in the G function, Eq. (3), we shall take, as in Ref. [15], the value $a(\mu) = -1.3$ for $\mu = 1.5$ GeV. Note that $\partial G(s)/\partial s$ does not depend on μ or $a(\mu)$.

2.2. Decay amplitude and invariant DK mass distribution in the $\bar{B}_s^0 \to D_s^-(DK)^+$ decay

We now discuss the amplitude for the decay $B_s^0 \to D_s^-(DK)^+$ decay, and its relation to the DK elastic scattering amplitude studied above. The basic mechanism for this process is depicted in Fig. 1, where, from the $\bar{s}b$ initial pair constituting the B_s^0, a $\bar{c}s$ pair and a $\bar{s}c$ pair are created. The first pair produces the D_s^-, and the DK state arises from the hadronization of the second pair. Let us consider in some more detail the hadronization mechanism. To construct a two meson final state, the $c\bar{s}$ pair has to combine with another $\bar{q}q$ pair created from the vacuum. We introduce the following matrix,

$$M = v\bar{v} = \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} & \bar{c} \end{pmatrix} = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} & u\bar{c} \\ d\bar{u} & d\bar{d} & d\bar{s} & d\bar{c} \\ s\bar{u} & s\bar{d} & s\bar{s} & s\bar{c} \\ c\bar{u} & c\bar{d} & c\bar{s} & c\bar{c} \end{pmatrix}, \tag{17}$$

which fulfills:

$$M^2 = (v\bar{v})(v\bar{v}) = v(\bar{v}v)\bar{v} = \left(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c\right) M\,. \tag{18}$$

The first factor in the last equality represents the $\bar{q}q$ creation. This matrix M is in correspondence with the meson matrix ϕ [48]:

$$\phi = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{\eta}{\sqrt{3}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & K^0 & D^- \\ K^- & \bar{K}^0 & \frac{\sqrt{2}\eta'}{\sqrt{3}} - \frac{\eta}{\sqrt{3}} & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}, \tag{19}$$

where a $\eta - \eta'$ mixing angle $\sin\theta = -1/3$ is assumed [49].

The hadronization of the $c\bar{s}$ pair proceeds then through the matrix element $(M^2)_{43}$, which in terms of mesons reads:

$$(\phi^2)_{43} = K^+ D^0 + K^0 D^+ + \cdots\,, \tag{20}$$

where only terms containing a KD pair are retained, since coupled channels are not considered in this work. We note that this KD combination has $I = 0$, as it should, since it is produced from a $c\bar{s}$, which has $I = 0$, and the strong interaction hadronization which conserves isospin (the $\bar{q}q$ with the quantum numbers of the vacuum has $I = 0$).

Let t be the full amplitude for the process $B_s^0 \to D_s^-(DK)^+$, which already takes into account the final state interaction of the DK pair. Also, let us denote by v the bare vertex for the same reaction. To relate t and v, that is, to take into account the final state interaction of the DK pair, as sketched in Fig. 3, we write [50, Sec. IIE]:

$$t = v + vG(s)T(s) = v(1 + G(s)T(s))\,. \tag{21}$$

From Eq. (1), the previous equation can also be written as:

$$t = v\frac{T(s)}{V(s)}\,. \tag{22}$$

It is interesting to briefly sketch here a derivation of Eq. (21), which is done in detail in Ref. [50]. The crucial point is that, in principle, the loop function appearing explicitly in Eq. (21) need not be the same than the one appearing implicitly in the T-matrix. The chiral unitary approach with a sharp cutoff in G is found equivalent to the use of a Schrödinger equation, using a potential with the form:

$$V(\vec{q}, \vec{q}\,')\big|_{S\text{-wave}} = V\,\theta(q_{\max} - |\vec{q}\,|)\,\theta(q_{\max} - |\vec{q}\,'|)\,, \tag{23}$$

which guarantees that the amplitude has the same form [38],

$$T(\vec{q}, \vec{q}\,')\big|_{S\text{-wave}} = T\,\theta(q_{\max} - |\vec{q}\,|)\,\theta(q_{\max} - |\vec{q}\,'|)\,. \tag{24}$$

Above, $q_{\max}$ is the cut off that regularizes the G function. It must be understood that $q_{\max}^{-1}$ is a measure of the range of the meson–meson interaction, a residual interaction at the quark level. In the loop shown in Fig. 3 we shall have $\theta(q_{\max} - |\vec{q}|)$ coming from the meson–meson T-matrix (shaded circle), but we also have an extra range factor from the production vertex (shaded square). However, this latter vertex comes from weak interactions and hadronization at the quark level (measuring interquark distances), which are of shorter range than the residual (Van der Waals like) meson–meson interaction. This means that we could account for it by means of a factor $\theta(\tilde{q}_{\max} - |\vec{q}\,|)$, with $\tilde{q}_{\max} > q_{\max}$. Then $\theta(\tilde{q}_{\max} - |\vec{q}\,|)\,\theta(q_{\max} - |\vec{q}\,|) = \theta(q_{\max} - |\vec{q}\,|)$ and the first loop becomes the same G function as in meson–meson scattering. Notice that the functions V and T on the r.h.s. of Eqs. (23) and (24) are the ones that would appear in Eq. (1). This clarifies what we mean by having a potential V energy dependent or independent, the energy dependence appearing explicitly in V on the r.h.s. of Eq. (23). Interestingly, Eq. (22) can also be derived from general principles (unitarity and analyticity), as done in Refs. [51,52] (see also Ref. [53]).[3] The extra factor $R(s)$ appearing in Eq. (34) of Ref. [51] is the equivalent of our v in Eq. (22). In this way, one does not

[3] It is worth stressing that Ref. [51] is the first work in which the hadronization mechanism from a scalar source (with vacuum quantum numbers) after the decay of a relatively heavy hadron is considered, something similar to our mechanism in Eq. (18).

have to use cutoffs and the range effects of a quantum mechanical formulation, among other things, are incorporated in counterterms that appear in this function $R(s)$.

Because of the presence of the bound state below threshold, this process will depend strongly on s in the kinematical window ranging from threshold to 100 MeV above it, so we can safely assume that t depends only on s. Hence, the differential width for the process under consideration is given by:

$$\frac{d\Gamma}{d\sqrt{s}} = \frac{1}{32\pi^2 M^2_{\bar{B}^0_s}} p_{D_s^-} p_K \, |t|^2 = \mathcal{C} p_{D_s^-} p_K \left| \frac{T(s)}{V(s)} \right|^2 , \tag{25}$$

where the bare vertex v has been absorbed in $\mathcal{C}$, a global (but otherwise not relevant) constant, and where p_K is given in Eq. (5) and $p_{D_s^-}$ is the momentum of the D_s^- meson in the rest frame of the decaying $\bar{B}^0_s$, given by:

$$p_{D_s^-} = \frac{\lambda^{1/2}(M^2_{\bar{B}^0_s}, M^2_{D_s^-}, s)}{2M_{\bar{B}^0_s}} . \tag{26}$$

3. Results

We want to investigate the presence of the $D^{*+}_{s0}(2317)$ state in the $(DK)^+$ scattering amplitude. In order to explore the sensitivity of the decay $\bar{B}^0_s \to (DK)^+ D_s^-$ to the presence of this bound state, we generate synthetic data from our theory for the differential decay width for the process with Eqs. (25) and (12). We generate 10 synthetic points in a range of 100 MeV starting from threshold. To each centroid, we assign the value obtained with the central values explained in Subsection 2.1 ($10^3\alpha = 2.06$ GeV^{-2}, $a(\mu) = -1.3$, and $M_{D^{*+}_{s0}} = 2317$ MeV). We shall study two different cases, in which each experimental point is given an error of a 5% or a 10% of the highest value of the differential decay width. Taking these synthetic data as experiment-given data, we perform the inverse problem of analyzing them with our theory [Eq. (25), together with Eqs. (1) and (10)]. Obviously, the reproduction of the data must be perfect, but we recall that the scope here is to investigate the experimental accuracy that is actually needed to obtain reliable values for the quantities fitted or predicted from our theory ($M_{D^{*+}_{s0}}$, a_0, r_0, and P_{DK}). The analysis of these synthetic data goes as follows. We generate around $2 \cdot 10^3$ sets of random experimental points, in which each centroid is varied around its theoretical value according to a Gaussian distribution with the error given to each point. For each of these sets of random points, the free parameters ($a(\mu)$, with μ fixed at 1.5 GeV, $M_{D^{*+}_{s0}}$, and α, plus the arbitrary normalization constant $\mathcal{C}$) are fitted to the data. After the whole run, a central range, containing a 68% of the values of the considered quantities (the differential decay width, the fitted parameters, and the predicted values) is retained. It is worth stressing here that, since the centroid of the experimental point in each set of random experimental points is varied, a good reproduction of the random synthetic data is quite natural but not completely trivial.

The generated synthetic data are shown in Fig. 4. As explained, they have two different error bars, the smaller one corresponding to a 5% experimental error and the larger one to a 10%. As commented above, they exactly match the central curve (dash-dotted line) produced with the central parameters of the theory. A solely phase space distribution (*i.e.*, a differential decay width proportional to $p_{D_s^-} p_K$, but with no other kinematical dependence of dynamical origin) is also shown in the figure (dashed line). The first important information to be extracted from the figure is that the data are clearly incompatible with this phase space distribution. This points to the presence of a resonant or bound state or,

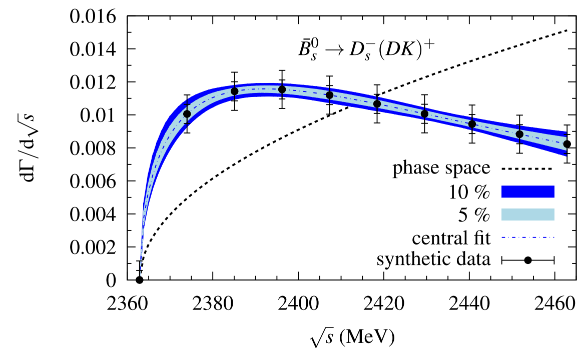

Fig. 4. Differential decay width for the reaction $\bar{B}^0_s \to D_s^-(DK)^+$. The synthetic data (generated as explained in the text) are shown with black points. The smaller (larger) error bars correspond to a 5% (10%) experimental error. The dash-dotted line represents the theoretical prediction obtained with the central values of the fit. The light (dark) bands correspond to the estimation of the error (by means of a MC simulation) when fitting the data with 5% (10%) experimental error. The dashed line corresponds to a phase space distribution normalized to the same area in the range examined.

Table 1
Fitted parameters (α, $M_{D^{*+}_{s0}}$ and $a(\mu)$) and predicted quantities ($|g|$, a_0, r_0, P_{DK}) for $\mu = 1.5$ GeV. The second column shows the central value of the fit, whereas the third (fourth) column presents the errors (estimated by means of MC simulation) when the experimental error is 5% (10%).

	Central Value	5 %	10 %
$10^3\ \alpha$ (GeV^{-2})	2.06	$^{+0.17}_{-0.40}$	$^{+0.10}_{-1.09}$
$M_{D^*_{s0}}$ (MeV)	2317	$^{+14}_{-24}$	$^{+21}_{-73}$
$a(\mu)$	-1.30	$^{+0.15}_{-0.37}$	$^{+0.27}_{-0.49}$
$\lvert g\rvert$ (GeV)	11.0	$^{+1.0}_{-0.6}$	$^{+2.2}_{-1.1}$
a_0 (fm)	-1.0	$^{+0.2}_{-0.2}$	$^{+0.4}_{-0.5}$
r_0 (fm)	-0.14	$^{+0.06}_{-0.03}$	$^{+0.16}_{-0.04}$
P_{DK}	0.75	$^{+0.07}_{-0.06}$	$^{+0.16}_{-0.11}$

at least, to some strong final state interactions. Two error bands are shown in the same figure, the lighter and smaller (darker and larger) one corresponding to a 5% (10%) experimental error. The fitted parameters ($a(\mu)$, $M_{D^{*+}_{s0}}$, and α) are shown in Table 1, together with their errors[4] (the constant $\mathcal{C}$, which absorbs the production vertex v, is only linked to the arbitrary normalization of Fig. 4, and hence it is not shown in Table 1). Note that, with a 5% experimental error, we get $M_{D^{*+}_{s0}} = 2317^{+14}_{-24}$ MeV, and if the error is increased to 10%, the value is $M_{D^{*+}_{s0}} = 2317^{+21}_{-73}$ MeV. Here we are mainly concerned with the upper error, in the sense that it is the one that defines if the bound state is clearly below the DK threshold (which is slightly above 2360 MeV) or not. Considering this error, we see that the mass obtained is well below the threshold, at the level of 2σ (3σ) for the case of a 5% (10%) experimental error. This is a good information: experimental data with a 10% error, which is clearly feasible with nowadays experimental facilities, can clearly determine the presence of this below threshold state $D^{*+}_{s0}(2317)$.

We can also determine P_{DK}, the probability of finding the DK channel in the $D^{*+}_{s0}(2317)$ wave function. It is shown in the last row of Table 1. As stated, the central value $P_{DK} = 0.75$ is the same

[4] To avoid unphysical values of the fitted parameters $a(\mu)$ and α, which could numerically reproduce each set of the randomly generated experimental points, they are restricted to vary within a sensible range, but making sure that this range is larger than the error obtained for these two parameters and shown in Table 1.

as the initial one, but we are here interested in the errors, which are small enough even in the case of a 10% experimental error. This means that with the analysis of such an experiment one could address with enough accuracy the question of the molecular nature of the state ($D_{s0}^{*+}(2317)$, in this case).

Finally, it is also possible to determine other parameters related with DK scattering, such as the scattering length (a_0) and the effective range (r_0). They are also shown in Table 1. They are compatible with the lattice QCD studies presented in Refs. [29,30]. Namely, the results from Ref. [30] are shown in Eqs. (16), and their mutual compatibility is clear.

4. Conclusions

In the present work we have selected a reaction which is Cabbibo favored, the $\bar{B}_s^0 \to D_s^-(DK)^+$ weak decay, and have looked at the DK invariant mass distribution from where we expect to obtain relevant information on the nature of the $D_{s0}^{*+}(2317)$. As an input to our theoretical prediction of this reaction, we have taken the experimental value of the mass of this state and information on the KD scattering amplitude from a recent lattice QCD analysis.

After predicting the differential width of this reaction, and since there are no actual data on this distribution, we have taken these theoretical results and we have selected a few points assuming that they are actual "experimental data", associating to them an "experimental error" of 5% or 10%. Then we have made a fit to these "synthetic data" in order to extract from there the KD scattering amplitude, above and below threshold. We prove that with both errors, typical of present experimental data of spectra in B decays, one can obtain the KD scattering amplitude with enough precision to predict that there is a KD bound state. We also predict the scattering length and effective range of the KD interaction and, very important, we show that we can predict, with relatively small error, the probability of the mesonic KD component in the wave function of the $D_{s0}^{*+}(2317)$ resonance. From the QCD lattice results one induces about 70% probability and we show that this number can be obtained from the analysis of the B decay spectra with sufficient precision to make the number significative of the main nature of the $D_{s0}^{*+}(2317)$ resonance as a basically KD molecular state with a smaller mixture of other components.

The study done here should stimulate the implementation of the experiment, for which we have made estimates of a relatively large branching fraction.

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